11.1-11.2 Parametric Curve Fundamentals

Generally,

$$\begin{cases} x = x_0 + ut \\ y = y_0 + st \end{cases}, \quad -\infty < t < \infty$$

is the parametric form of a line passing through (x_0, y_0) with slope $\frac{s}{u}$.

For circles,

$$\begin{cases} x = x_0 + R\cos t \\ y = y_0 + R\sin t \end{cases}, \quad 0 \leq t \leq 2\pi$$

Slope of tangent line can be found with:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

Area under curves can be found by integration:

$$A = \int_a^b y(t)\frac{dx}{dt}dt$$

Arc length: Can use a t-integral as long as familiar x, y integrals:

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\,dt$$

Surface Area: Similarly as for the length of the arc, the t -integrals can be used to solve the area of the surface of the revolution in addition to the x, y integrals:

$$\begin{aligned} A &= \int_a^b 2\pi y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \\ &= \int_a^b 2\pi g(t)\sqrt{f'(t)^2 + g'(t)^2}\,dt \end{aligned}$$

Revolving around a y-axis is similar, instead of including $f(t)$ in integration rather than $g(t)$.