1.2 – Gaussian Elimination

Matrix Example

$$A = \begin{bmatrix} 3 & 2 & -4 \\ 0 & 5 & 1 \end{bmatrix}, \quad a_{i,j} \text{ format}$$

Coefficient matrix includes only coefficients of variables in the system of linear equations. Moving the number items across the equals sign into the matrix side forms an 'augmented matrix' with another column.

Similar strategy as back-substitution – we want "1's" in the leading coefficient spots to reach row-echelon form.

$$\begin{bmatrix} x & x & \vdots & x \\ x & x & \vdots & x \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \vdots & x \\ 0 & 1 & \vdots & x \end{bmatrix}$$

(w/ ones in coefficient spots and zeros everywhere else on the left.)

Gaussian elimination finds REF,

Gauss-Jordan elimination finds <u>reduced</u> REF.

Ex.

$$2x_1 + 4x_2 - 2x_3 = 2$$
$$4x_1 + 9x_2 - 3x_3 = 8$$
$$-2x_1 - 3x_2 + 7x_3 = 10$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 4 & 9 & -3 & 8 \\ -2 & -3 & 7 & 10 \end{array}\right] \quad \begin{array}{l} R_1 + R_3 \to R_3 \\ -2R_1 + R_2 \to R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 12 \end{array}\right] \quad R_2 - R_3 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -4 & -8 \end{array}\right] \quad \begin{array}{l} R_1 / 2 \to R_1 \\ \\ R_3 / -4 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & +2 \end{array}\right]$$

Row-echelon form from Gaussian Elimination

$x_3 = +2$

$x_2 + x_3 = 4 \qquad x_2 = 2$

$x_1 + 2x_2 - x_3 = 1 \qquad x_1 = -1$

Sol. $= (-1, 2, 2)$

Note: Could have to find matrix in reduced REF.